Take Home Exam 03: Critical Radius for Nucleation of Water from the Vapor Phase

Assigned: 09/01/2022

Due (as pdf by email) 09/04/2022 (Sunday)

•you will receive a simple letter grade for your report

•You may submit your answers in one of two ways:

1) For typed answers: as a .docx file (as is) or converted into a pdf file. (DO NOT SEND GOOGLE DOC)

For handwritten answers: Please scan as images, and group together into one pdf file. Or you may hand them manually to my office (ECME-212)

HW 03.1

(i) *Give the units for each of the parameters in the following equations*. The subscripts refer to the "volume" and the "surface".

$$\Delta G_V = \Delta H_V - T\Delta S_V \qquad (1)$$

$$\gamma_S = \Delta G_S = \Delta H_S - T\Delta S_S \qquad (2)$$

If the vapor phase of H_2O as the reference state, then ΔH_V is the enthalpy of evaporation of water into the vapor (its value is 41 kJ mol^{-1}).

(ii) We wish to transform Eqns (1) and (2) and rewrite them on a per-atom basis:

for example ΔH_V, which normally has units of J mol^{-1}, becomes $\Delta H_V^{per\ atom} = \frac{\Delta H_V}{N_A}$, etc.

Write down the quantities in Eq. (2) on a per atoms basis, for example:

$\gamma_S^{per\ atom} = \frac{\gamma_S}{?}$ Hint: play close attention to the units (the surface quantities are related to the number of atoms per unit surface area - how would you estimate that number).

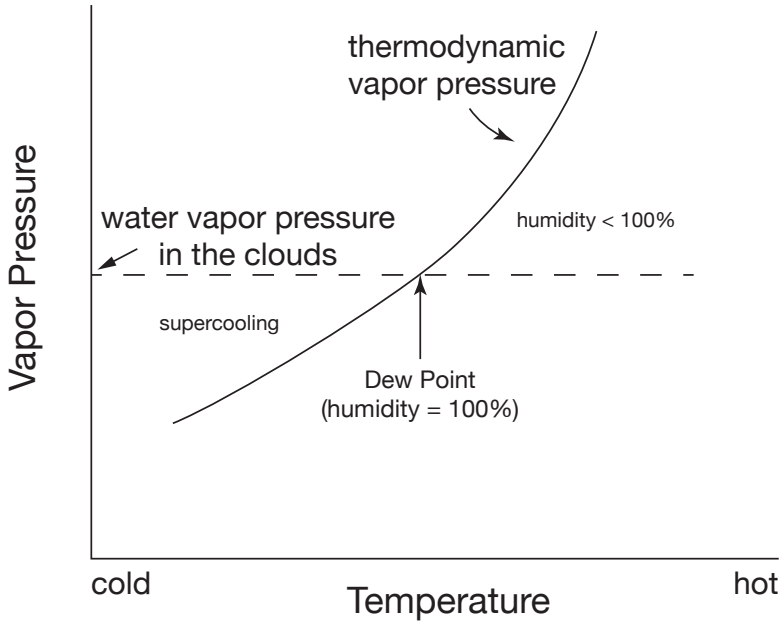
HW03.2

The surface energy for water is $\gamma_S = 0.072$ J m^{-2}. And ΔG_V with units of J m^{-3} is related to supercooling below the Dew Point by

$\Delta G_V = \Delta H_V \frac{\Delta T}{T_{DP}}$ as derived in class. We also derived that the critical radius for nucleation is given by

$$r^* = \frac{2\gamma_S}{\Delta G_V}$$

The Dew Point is the temperature where the atmospheric vapor pressure becomes equal to the thermodynamic vapor pressure of water. As shown in the graph below if the atmospheric vapor pressure is less than the thermodynamic vapor pressure, the humidity is less than 100%. As the temperature in the environment falls the thermodynamic vapor pressure becomes equal to the atmospheric pressure and the Dew Point is reached.



In the following analysis you may assume that the Dew Point is 15 ºC or 288 K.

$\Delta H_V = 41$ kJ mol^{-1}. (remember to convert into J mol^{-1}).

You are asked to make a plot of r^* as a function of ΔT (the supercooling) for a range of 1 to 15K.

Make another plot where r^* is plotted on a $\log_{10}$ scale, while the ΔT remains on a linear scale.

Pick one value of r^* and calculate the number of atoms (actually water molecules) in the embryo of critical size.